6.3/26)

$$\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx = \int \frac{1}{u}\,du = \ln|u|+C = \ln|e^x-e^{-x}|+C$$

Let $u=e^x-e^{-x}$

$du=(e^x+e^{-x})\,dx$

Alternatively...

$$\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$$

Let $u=e^x$

$du=e^x\,dx$

$$= \int \frac{e^x+e^{-x}}{e^x-e^{-x}}\cdot\frac{e^x}{e^x}\,dx \Rightarrow \int \frac{u+\frac{1}{u}}{(u-\frac{1}{u})(u)}\,du = \int \frac{u+\frac{1}{u}}{u^2-1}\,du$$

$$\int \frac{u^2+1}{u(u^2-1)}\,du$$

27) $\int \frac{e^x}{1+e^{2x}}\,dx \Rightarrow \int \frac{1}{1+u^2}\,du$

$u = 1+e^{2x}$

$du = 2e^{2x}\,dx$

$u = e^x$

$du = e^x\,dx$

$e^{2x} = (e^x)^2$

$\tan^{-1}(u) + C$

$= \tan^{-1}(e^x) + C$

$$\frac{1}{2}\int \frac{2\,dx}{\sqrt{1-4x^2}}$$

$$\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}(u) + C$$

$u^2 = 4x^2$

$\therefore$ let $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$= \frac{1}{2}\sin^{-1}(u) + C$

$= \frac{1}{2}\sin^{-1}(2x) + C$

25) $\int x^2 e^{-2x^3}\,dx \Rightarrow -\frac{1}{6}\int 1\,du$

$u = e^{-2x^3}$

$du = -6x^2 e^{-2x^3}\,dx$ $\quad = -\frac{1}{6}u + C$

$-\frac{1}{6}du = x^2 e^{-2x^3}\,dx$ $\quad = -\frac{1}{6}e^{-2x^3} + C$

21)
$$\frac{1}{8}\int \frac{8x}{(4x^2+1)^3}\,dx \Rightarrow \frac{1}{8}\int \frac{1}{u^3}\,du$$

$u = 4x^2+1$

$du = 8x\,dx$

$$= \frac{1}{8}\int u^{-3}\,du$$

$$= \frac{1}{8}\,\frac{u^{-2}}{-2} + C$$

$$= -\frac{1}{16}(4x^2+1)^{-2} + C$$

23)

$$\int e^{\sin x} \cos x \, dx = \int 1 \, du$$

$u = e^{\sin x}$

$du = \cos x \, e^{\sin x}$

$= u + C$

$= e^{\sin x} + C$

$u = \sin x$

$du = \cos x \, dx$

$\int e^u \, du = e^u + C$

$= e^{\sin x} + C$

19)

$$\frac{1}{3}\int \frac{3x^2}{\sqrt{x^3+1}}\,dx \Rightarrow \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$$

$u = x^3+1$

$du = 3x^2\,dx$

$$= \frac{1}{3}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{3}\,\frac{u^{1/2}}{1/2} + C$$

$$= \frac{1}{3}\,\frac{(x^3+1)^{1/2}}{1/2} + C$$

$$\int \frac{x^2}{\sqrt{x^3+1}}\,dx$$

$u = \sqrt{x^3+1} = (x^3+1)^{1/2}$

$du = \frac{1}{2}(x^3+1)^{-1/2}(3x^2)\,dx$

$\frac{2}{3}du = (x^3+1)^{-1/2}(x^2)\,dx$

$$\frac{2}{3}\int 1\,du$$

$$= \frac{2}{3}u + C$$

$$= \frac{2}{3}\left(\sqrt{x^3+1}\right) + C$$

32)

$$\frac{1}{2}\int \cos^3(2t)\sin(2t)\,2\,dt$$

Let $u = 2t$

$du = 2dt$

$$\frac{1}{2}\int \cos^3 u \sin u\, du$$

Let $v = \cos u$

$dv = -\sin u\, du$

$-dv = \sin u\, du$

$$-\frac{1}{2}\int v^3\, dv$$

$$-\frac{1}{2}\left(\frac{v^4}{4}\right) + C =$$

$$= -\frac{1}{2}\left(\frac{(\cos u)^4}{4}\right) + C$$

$$= -\frac{1}{2}\left(\frac{[\cos(2t)]^4}{4}\right) + C$$

Let $u = \cos(2t)$

⋮

41) $\int \frac{dx}{e^x}$:($= \int \frac{e^x\,dx}{e^x \cdot e^x}$

$u = e^x$

$du = e^x\,dx$ $\qquad = \int \frac{du}{u^2}$

. . .

$\int e^{-x}\,dx$

$u = -x$

⋮

2b)

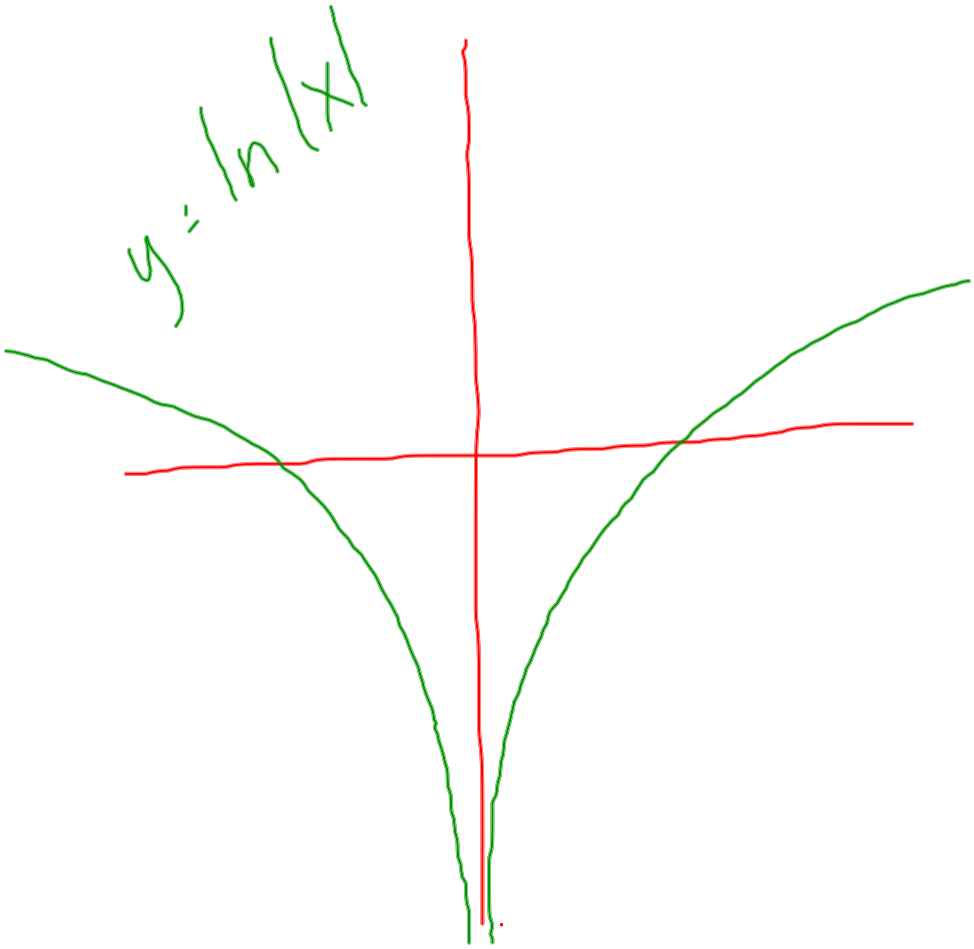
$$\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx \Rightarrow \int \frac{1}{u}\,du$$

$u = e^x - e^{-x}$

$du = (e^x + e^{-x})\,dx$

$= \ln|u| + C$

$= \ln|e^x - e^{-x}| + C$



17)

$$\frac{1}{14}\int 14t\sqrt{7t^2+12}\,dt$$

$u = 7t^2+12$

$du = \underline{14t\,dt}$

$$\frac{1}{14}\int \sqrt{u}\,du$$

$$= \frac{1}{14}\int u^{1/2}\,du =$$

$$\frac{1}{4}\frac{u^{3/2}}{3/2}+C = \frac{1}{14}\frac{(7t^2+12)^{3/2}}{3/2}+C$$

27) $\int \frac{e^x}{1+e^{2x}}\,dx \Rightarrow \int \frac{1}{1+u^2}\,du$

$u = e^x$

$du = e^x\,dx$

$e^{2x} = (e^x)^2$

$= \tan^{-1} u + C$

$= \tan^{-1}(e^x) + C$

(hint: Apply trig identity)

$$\int \sec^4 3\theta \, d\theta$$

=int(sec^2(3theta) [1+tan^2(3theta)]